Learning by Comparing: Effects of Direct Instruction, Discovery and Delay on Analogical Transfer

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Abstract

Previous research on analogy suggests that comparison between two instructional examples promotes analogical transfer and that direct instruction and inquiry based learning have their own advantages and disadvantages. The present study describes two experiments, the first designed to test the efficacy of directly instructing participants about common relational structure between analogous problems and the second designed to explore the effect of temporal delays on successful analogical transfer. Subjects' analogical reasoning was operationalized as their ability to solve a relationally similar target problem which required mapping of relevant concepts from the two base problems. We hypothesized that (i) analogical transfer would be independent of pedagogy, (ii) comparison between analogs would yield stronger solution schemas and that (iii) a delay would result in higher rates of solving the transfer than no delay. Results showed that (i) comparison did not yield better schema quality than no comparison (ii) analogical transfer did not depend on instructional method and (iii) delay fostered more efficient analogical transfer than no delay.

Learning by Comparing: Effects of Direct Instruction, Discovery and Unconscious Thought on Analogical Transfer

One cardinal goal of education especially in the context of problem solving is to foster analogical transfer because transfer follows from analogical reasoning, which is central to learning of abstract concepts (Richland, Holyoak & Stigler, 2004). Analogical transfer is the ability to apply knowledge to different situations, contexts or tasks, where that knowledge is applicable to the end that it helps solve a similar problem or deal with a new task. However, students often fail to transfer knowledge to analogous situations that they should have been able to correctly map deep structural knowledge to, what Alfred Whitehead (1929) referred to as the 'Inert Knowledge' problem. He believed that theoretical ideas should always find applications within students' curriculum but also recognized that this doctrine was very hard to implement because the central problem that educators face is designing a pedagogical approach that fosters students' acquisition of knowledge in such a way that it is applicable to novel situations therefore protecting new knowledge from becoming passive or inert.

A primary purpose of instructional science has been to address the question - What instructional manipulations can help students learn in a way that will optimize analogical transfer? Gentner (1983) defines an analogy as a comparison in which relational predicates, but few or no object attributes can be mapped from 'base' to 'target.' A relational predicate is the part of a sentence that indicates what is being said about the subject. e.g. in the following sentence, the italicized part is the predicate and the rest is the subject: 'Coldstone's ice cream cakes *are the most delicious and unhealthy cakes in this part of town*'. Gentner states in her paper a simple analogy: '*the hydrogen atom is* *like our solar system*' and explains how it should be interpreted based chiefly on the relational structure (structural similarities) of the two entities as opposed to superficial similarities (surface commonalities). A *base* analog can be thought of as a story/situation that is used to abstract deep structural concepts from and the *target* analog is a relationally similar story/problem to which structural concepts from the base can be transferred in order to solve it. The intended inferences in the previous example were: 'the electron *revolves around* the nucleus just as the planets *revolve around* the sun,' which talks about the deep structure of the *base* analog (hydrogen atom) and maps this structure to the *target* analog (solar system). A misinterpretation of this analogy would be: 'the hydrogen nucleus is *yellow and massive* just like the sun' because this observation is based solely on surface similarities.

Analogical reasoning involves the ability to perceive underlying structural similarities among corresponding objects that are superficially dissimilar (Holyoak & Thagard, 1995). For example if a child recognizes the structural similarity between a bee-hive and dog-house and is able to point out the functional correspondence between a dog living in a dog house and a bee living in a bee-hive. However, if the child got distracted by surface features such as the opening to the dog-house and the opening to the bee-hive, they would have missed the underlying conceptual analogy that makes the bee-hive so similar to the dog-house. Another example that illustrates the use of analogical reasoning is the often used analogy in physics between Coulomb's law and Newton's law of gravitation. Though both laws are inverse square force laws, there are stark differences between their surface features. Coulomb's law is used to measure the electrical force acting between two charges whereas Newton's law of gravitation measures the gravitational force between two masses. These laws can be said to be analogous because they both involve the same mathematical function though electricity and gravity are radically different domains (Podolefsky & Finkelstein, 2006). 'Analogical Encoding,' a term first introduced in Gentner, Loewenstein and Thompson's (2003) paper on learning and transfer is different from standard analogical reasoning because the applicability of the use of analogies in analogical encoding is limited to cases for which the learner already understands an appropriate source domain. Analogical encoding is thus defined as learning by drawing comparisons across examples.

In the past 30 years, researchers have elucidated many practices that have empirically been demonstrated to foster analogical transfer. Such strategies include self-explanation (Chi, De Leeuw, Chiu, & LaVancher, 1994; Siegler & Chen, 2008), spaced practice, and the use of both abstract and concrete representations to foster learning (Koedinger, Alibali & Nathan, 2008; Goldstone & Son, 2005). More recently, mounting evidence has suggested that comparing examples in order to abstract deeper structural commonalities is among the practices that are proven to promote students' robust learning (Gentner & Namy, 2006).

The facilitative effect on learning from comparing two isomorphic examples, or, comparison-driven learning, was first discovered by Gick and Holyoak (1983). In their study, Gick and Holyoak gave college undergraduates and high school students opportunities to study short story problems, such as The General story (an example of this story is included in the appendices). Students studied this problem, and its solution, and then attempted to solve a transfer problem in a semantically dissimilar context (the Tumor problem, also included in the appendices) that, unbeknownst to participants, could be solved in an analogous way to the General story: that is, by dividing a large resource into smaller units, the resource could be distributed around a central target and then applied simultaneously to eradicate that target (the convergence schema). The question was - what instructional conditions would lead students to spontaneously apply a learned schema to a novel situation?

Gick and Holyoak (1983) tried various manipulations in order to foster spontaneous analogical transfer of the convergence schema. For instance, along with a single analog, they provided students a general statement of the problem solution and in another study a diagram of the convergence schema in addition to the solution. Neither of these strategies produced successful transfer of the schema significantly above that of students who did not receive such instructional aids. Eventually, Gick and Holyoak had students compare two analogs before attempting the Tumor problem. Surprisingly, when students compared two superficially dissimilar analogs with the same common structure, they were more likely to solve the transfer task than when they compared two superficially similar analogs with the same common structure. Moreover, instructional manipulations that had initially failed to produce robust transfer of the convergence schema with one example now proved to be effective when they were incorporated with the provision of two examples. Gick and Holyoak thus had discovered the powerful effects that comparison-driven instruction can have on learning.

Gentner and her colleagues have since done work to demonstrate the robustness of this effect. For instance, when college students in an introductory business class studied a negotiation strategy by comparing the similarities of two examples that utilized the strategy, they were more likely to employ the negotiation tactic on a transfer test than students who studied each example separately (Kurtz, Miao & Thompson, 2001). Similar effects for comparison-driven learning have been demonstrated with preschool-age children's spatial reasoning (Colhoun, Gentner & Loewenstein 2008), abstract categorical reasoning (Kotovsky & Gentner, 1996), word learning (Gentner & Namy 1999; Namy & Gentner, 2002), and learning of animal defense mechanisms (Brown & Kane 1988).

Catrambone and Holyoak (1989) found that analogical transfer/mapping can be obtained even without hints about the base analog and target being related as long as subjects explicitly compare the analogs; although there is no pre-hint transfer when source analog and target problems are presented in different contexts. Pre-hint transfer in this case refers to the successful mapping of relational concepts from one analog to the other before a hint, that subtly expresses the similarity between the two situations, is presented. To enhance pre-hint transfer, Catrambone and Holyoak suggested that (i) target problems be reworded to emphasize structural features they share with the source analogs (ii) three source analogs be presented instead of two and (iii) detailed questions about problem schema be presented to subjects to help them focus on relevant aspects of the stories. Their study revealed that analogical transfer could occur within the same domain even without the provision of a hint but that inter-domain transfer (transfer between two unrelated and superficially dissimilar domains) depended on the presentation of a hint and explicit instructions (direct instruction) to use the analogs in order to solve the target problem. This study contributed to the literature on comparison driven learning by providing evidence for a correlation between strong schemas (assessment of similarity descriptions of analogs followed by comparison) and successful transfer.

Analogical Transfer and Instructional Guidance

One of the many questions in educational research that is still a subject of ongoing debate is *how* comparison should be facilitated. Should teachers allow students to abstract important relationships on their own or should they provide strong support and scaffolding in aiding students' abstractions? Is it possible to quantify the right amount of direct instruction that would serve to minimize students' stumbling on their quest to finding a solution but simultaneously present the same context as an inquiry based learning approach? These questions are closely related to what Koedinger & Aleven (2007) refer to as "The Assistance Dilemma" in research on cognitive tutors. How can we balance information/assistance giving and withholding in a learning environment to achieve optimal student learning? Providing guidance may ensure efficient communication and minimize floundering on the part of the student whereas withholding guidance may engage long-term memory and simulate the unguided context of a transfer situation (Koedinger & Aleven, 2007). Although addressing this question is crucial to translating educational research to classroom settings, very few studies have assessed the relative effectiveness of providing and withholding guidance during the comparison-process.

Across all studies on analogical transfer, students have always been prompted to compare the similarities of analogs by generating the deep structural commonalities on their own. One danger of such an approach is that students may abstract the wrong commonalities between similar analogs, such as irrelevant deep features. Indeed, Gick and Holyoak (1983) as well as Catrambone and Holyaok (1989) found that the quality of schemas (measured by the number of components of the convergence solution correctly identified) produced by subjects during comparison was significantly correlated with their later success at solving a transfer problem. One way around this difficulty is to provide students with sufficiently detailed comparisons that highlight deep common features of each example through means of direct instruction. Providing direct instruction regarding the commonalities that are most important across instructional examples has the potential to a) minimize errors in abstracting irrelevant schemas, b) make relevant commonalities more salient, and c) lead to more efficient learning (with regards to time on task). It remains to be seen, however, whether directly instructing students to compare relevant schemas could actually produce these effects on learning.

Though the constructivist notion that discovery learning is the best way to prompt deep and lasting understanding of scientific concepts is widely accepted and popular in the education community, pragmatically it is quite apparent that most of what students know about science was taught to them, rather than discovered by them. Proponents of discovery learning agree with the Piagetian claim that each time we prematurely teach a child something he could have discovered for himself, we're keeping that child from inventing it and thus fully understanding it (Piaget, 1970, p. 715). But most developmental and cognitive theories recognize that children in discovery situations are more likely than those in direct instruction environments to encounter misleading feedback, make encoding errors and causal misattributions, and to experience incompetency in the domain/concept being taught (Mayer, 2004).

Klahr and Nigam (2004) designed a study to measure the relative effectiveness of discovery learning and direct instruction during the initial learning process as well as during transfer to a novel analogous situation. To evaluate this prediction, they created examples of both discovery learning and direct instruction and finally tested learners with an appropriate transfer task. The context in which the two instructional approaches were contrasted was an important science objective called *control of variables strategy* (CVS), which is a procedural method for creating experiments in which a single contrast is made between experimental conditions. CVS is the basic strategy that enables children to design un-confounded experiments from which valid causal inferences can be made.

Klahr and Nigam (2004) tested the following three hypotheses: (i) Direct Instruction is more effective than discovery learning in teaching children CVS (ii) When evaluating science fair posters, children who have mastered CVS outperform those who have not and (iii) If children achieve mastery of a new procedure then the way that they reached that mastery will not affect their ability to transfer what they have learned. Results revealed that more children achieved mastery of CVS through direct instruction than discovery learning and when asked to assess science fair posters and make scientific judgments, children who had mastered CVS either via discovery or via DI (direct instruction) performed equally well thus providing evidence for path independent transfer. This study contributed to the wealth of scientific literature challenging the conjectured superiority of discovery approaches in teaching young children the basics of scientific investigation. Whether direct instruction and discovery learning are equally effective in the context of adult populations remains relatively unexplored and will be investigated in this paper.

Analogical Reasoning, Temporal Delay and Conscious vs. Unconscious Thought

Another area within the domain of analogical encoding and transfer, which remains relatively unexplored, is the effect of temporal delays on analogical mapping. Spencer and Weisberg (1986) found that delays and contextual changes eliminated transfer unless a hint was provided. Catrambone and Holyoak (1989) furthered this research by providing subjects with two analogs and asking them to write summaries about the stories comparing similarities. This was followed by a 30 min distracter survey which served as a change of context. Note that what Catrambone and Holyoak termed 'change of context', can also be perceived as a chance for unconscious thought since the survey they filled out before the transfer problem is essentially a buffer task between the analogs and transfer problem. Since their results replicated Spencer and Weisberg's results, they found that spontaneous transfer is not produced without the provision of a hint.

An intriguing insight gained from Catrambone and Holyoak's study is that some students who failed to transfer their knowledge spontaneously were able to do so after a hint was presented. This suggests that features of the convergence problem were stored in the subjects' long term memories, which is why they were able to recall and apply them to the transfer problem when given instructions to after a week's delay. It follows that the features of the convergence solution were not recalled from subjects' working memories since the average duration of attention given to information in working memory is approximately 18 to 20 seconds (Peterson & Peterson, 1959). This knowledge would have been lost and never been retrieved, had it not been encoded in subjects' long term memories. And since they were not consciously aware of possessing this knowledge, a hint was what triggered this knowledge to become available to their conscious thought leading to successful recall.

When target problems are reworded such they convey one of the solution relevant features of prior analogs, subjects who are given explicit comparison instructions write appropriate and strong schema for the base analogs and show improved spontaneous transfer (Gick & Holyoak, 1987; Keane, 1985; Catrambone & Holyoak, 1989). Brown, Kane, and Echols (1986) as well as Brown (1989) found that directing subjects to compare the source analogs on problem oriented dimensions while varying superficial features fosters schematic representations that are able to support transfer across contextual changes.

One possible reason why subjects who wrote better schemas produced the convergence solution prior to hints more often than subjects who didn't write good schemas is because rewording the target problem to cue subjects in to the solution-relevant features of the base analogs can be interpreted as being equivalent to giving them a hint or giving them subtle instructions to use the prior analogs to solve the target. This particular strategy used across the aforementioned studies was a very efficient manipulation since it eliminated demand characteristics because the subjects did not need to receive explicit hints anymore; they were prompted to progress in the right direction through the rewording of the problem itself. We will be employing a similar manipulation in both the studies described in this paper precluding the use of hints.

Another inconspicuous and discreet manipulation that we plan on using concerns temporal delays which as described earlier can be viewed as opportunities for unconscious thought. Dijksterhuis and Nordgren (2006) define conscious thought as object-relevant or task-relevant cognitive processes that occur while the object or task is the focus of one's conscious attention e.g. if I consciously compare Whistler, Canada and

Livigino, Italy as ski trip destinations, after some thinking I might realize that 'The Livigino valley is a better destination because it is tax free whereas Whister is better because it's closer to the US.' Unconscious thought on the other hand refers to object relevant cognitive processes that occur while conscious attention is directed elsewhere e.g. after struggling to choose between Whistler and Livigino, I might stop consciously thinking about my pending decision and when I'm driving down to the movies 24hrs later, trivia about Italy I heard on Discovery Channel 5 years ago such as "Livigino is known for it's famous cheese: Semigrasso d'Alpe" might surface into my consciousness and tip my preference in favor of Italy.

The context that we are examining conscious/unconscious thought in our study is different from the way Dijksterhuis and his colleagues looked at it because in their research they explored the advantages of unconscious thought in the process of decision making (Bos, Dijksterhuis, van Baaren, 2007) whereas in this paper we will be exploring unconscious and conscious thought mechanisms in the context of problem solving specifically, analogical transfer. It is however important to note that in our study as well as Dijksterhuis' research, the transition from indecision to a preference for variable A over variable B or the transition from not being able to see structural commonalities to complete analogical mapping between two functionally dissimilar stories, is the result of unconscious thought although the final thought itself is conscious.

Dijksterhuis and Nordgren view *attention* as the primary distinguishing factor between unconscious thought and conscious thought because conscious thought can be seen as thought with attention whereas unconscious thought is thought without attention. Interestingly conscious thought does not comprise only conscious processes in fact it cannot take place without unconscious processes being active at the same time e.g. walking is a conscious activity but various unconscious processes such as those responsible for pace of walking, where to step, maneuvering way through crowds, stopping at lights, giving way to others, must be activated in order for us to walk properly.

The questions being explored in this paper have important implications for education at various levels. In general this study will shed light on what kinds of instructional strategies promote efficient analogical transfer. If direct instruction results in better rates of problem solving than discovery learning, then institutions from the elementary to graduate level would benefit from promoting direct instruction in classroom settings as opposed to letting students come to the same scientific realization through trial and error methods or employing exploration and discovery strategies. Additionally if unconscious thought along with conscious thought yields better rates of problem solving than conscious thought alone, this could imply that informing students that they will be tested on similar problems (providing an ambiguous goal) and giving them more time to unconsciously process the educational material they will be tested on before actually testing them, might be a more efficacious way of promoting learning compared to testing students immediately after instruction.

The Present Experiments

The primary goals of the two experiments reported in this paper are (a) to assess differences in analogical transfer for comparison and no comparison conditions, (b) to gauge the differences in analogical transfer for both direct instruction and discovery learning approaches and (c) to explore the effects of temporal delays (chance for unconscious thought) on problem solving. Experiment 1 was designed to assess the relative contributions of comparison and direct instruction on analogical transfer. A graphical depiction of this 2 (Comparison: Compare/No Compare) x 2 (Instructional method: Direct Instruction/Discovery) design can be viewed in Table 1 where the columns indicate the type of instruction subjects received during the reading phase and the rows indicate whether or not subjects were asked to compare the two analogs in the same phase.

Within the Table, Cells A vs B illustrate the typical design in studies aimed at assessing comparison-driven learning – for example, Gentner (1983) contrasted students who received negotiation strategies on the same study page and were asked to compare their commonalities (Cell A) with students who saw negotiation strategies on separate study pages and were asked to summarize them (Cell B). Cell C is what is typically employed in instructional studies examining the effects of Direct Instruction on learning that include comparison between good and bad examples and explanations of what makes an example good or bad (e.g. Klahr & Nigam, 2004). Finally, Cell D is what has, to our knowledge, never been examined in the literature in the instructional or cognitive sciences. Thus, the primary question being addressed in Experiment 1 is whether providing direct instruction with regards to two problems' commonalities can produce a similar effect on students' learning when compared to having students abstract commonalities on their own.

Experiment 2 addresses the effect of time delay on analogical transfer. This study investigates whether thinking about the relational commonalities of an analog through goal directed unconscious thought results in better mapping of concepts to the transfer problem than allowing only conscious thought for analogical transfer? In other words, would receiving a 5 minute delay, during which participants solve a non analogous problem, before solving the target analog result in better rates of problem solving compared to receiving no delay? Table 2. represents this 2 (Comparison: Compare/No Compare) x 2 (Instructional method: Direct Instruction/Discovery) x 2 (Time Delay: Delay/No Delay) design in a tabular format.

We predict that in Experiment 1. (i) analogical transfer will be strong for both Comparison conditions and (ii) Direct instruction and Discovery will be equivalent in their effects on learning such that analogical transfer is path independent and for Experiment 2. in addition to the first two hypotheses, (iii) delay will lead to a higher rate of solving transfer problems than no delay.

	Discovery	Direct Instruction
Comparison	A	D
No Comparison	B	C

Table 1. Design of the present experiment: The columns show the type of instruction subjects received during the reading phase and the rows indicate whether or not subjects were asked to compare the two analogs in this phase. Gentner and her colleagues examined the effect of contrasting cells A and B (Gentner, 1983; Gentner et al., 2003; Gentner & Namy, 2006) and showed that cell A produces better analogical transfer than cell B (i.e. demonstrating the effect of comparison-driven learning). Cell C is what is typically done in studies that are aimed at assessing effects of direct instruction (often, studies with Cell C are contrasted with Cell B) (e.g. Klahr & Nigam, 2004). Cell D has, to our knowledge, never been examined before.

Study 1

Method

Participants

This study consisted of 33 Carnegie Mellon Undergraduates who received one research credit for their participation. Each student was randomly assigned to one of four conditions, so that there were approximately 8 participants in each condition.

Materials

All study materials were presented to subjects after being typed up on paper. In total, subjects read two analogous stories - Fire Chief and the General stories (see Appendix A), they then summarized or compared the two analog stories according to their condition, after which they attempted to solve one distractor problem (the Birthday Party Problem). Finally, they attempted to solve a transfer problem (the Tumor problem) which was analogous to the first two base analogs (see Appendix B).

Design

This experiment utilized a 2(Compare/No compare) x 2(Direct instruction vs. Discovery learning) factorial design with comparison and instructional method both as between subject factors (See Table 2.). The dependent variable was performance on the Tumor Problem and it was scored as a binary variable - by whether or not subjects were able to correctly or incorrectly identify and list the convergence solution described below.

Procedure

Subjects in all conditions received two analogous stories (the Fire Chief and General stories, in a counterbalanced order), one distracter problem (the Birthday Problem), and one transfer problem (The Tumor Problem). All three analogous problems (the Fire Chief, General, and Tumor problems) can be solved by dividing an excess resource into smaller units, surrounding a central target, and converging on the target all at once.

A prior calibration of these materials with 31 undergraduates in the Fall of 2009 revealed that 44% of students solved the Fire Chief problem and 44% solved the General problem spontaneously, while only 14% of undergraduates successfully solved the Tumor problem. Due to the unequal rates of solving these three problems, instead of employing a fully counterbalanced design across problem types, we have always included the most difficult problem – the Tumor problem – as the transfer task, and other two problems – the Fire Chief and General stories – as the two analogous stories that subjects study before attempting the transfer problem.

Subjects were tested in a quiet testing room on campus and were told that they would be participating in a study about reading comprehension. After subjects gave their consent, they were told that they would be given five minutes to read and study a story (the General or the Fire Chief story, counterbalanced across participants). The table below outlines what happens across the entire experimental session.

Table 2. Outline of Experiment 1

Condition	Read Analog 1	Summarize	Read Analog 2	Summarize	Compare	Solve Distracter	Solve Transfer
Direct Instruction No Compare	Read 5 mins	Study provided summary 3 mins	Read 5 mins	Study provided summary 3 mins	N/A	Solve 5 mins	Solve 5 mins
Discovery No Compare	Read 5 mins	Generate summary 5 mins	Read 5 mins	Generate summary 5 mins	N/A	Solve 5 mins	Solve 5 mins
Direct Instruction Comparison	Read 5 mins	N/A	Read 5 mins	N/A	Study provided comparisons 6 mins	Solve 5 mins	Solve 5 mins
Discovery Comparison	Read	N/A	Read	N/A	Generate comparisons	Solve	Solve

	5 mins		5 mins		10 mins	5 mins	5 mins

The experiment had two phases: The Reading phase and the Solution generation phase. The Reading phase was different for each subject depending on their condition:

Direct Instruction/No Compare: In this condition, students first read the first analog problem (either the General or Fire Chief problem) for 5 minutes. Then, they were asked to read a summary of the problem (see appendix) for 3 minutes. Students then read the second analog problem (either the General or Fire Chief problem) for 5 minutes, and afterwards, read a summary of the problem for 3 minutes.

Disovery/No Compare: In this condition, students initially read the first analog problem (either the General or Fire Chief problem) for 5 minutes. They were then given 5 minutes to fill out a table that prompted them to summarize the main parts of the story (see appendix A). Students then read the second analog problem (either the General or Fire Chief problem) for 5 minutes, and afterwards, were given 5 minutes to fill out a table that prompted them to summarize the main parts of the story.

Direct Instruction/Comparison: In this condition, students read the first analog problem (either the General or Fire Chief problem) for 5 minutes. Then, they proceeded to read the second analog problem for 5 minutes. After this, subjects were given 6 minutes to read a summary of both stories, presented side by side in tabular format.

Discovery/Comparison: In this condition, students read the first analog (either the General or Fire Chief problem) for 5 minutes. Then, they proceeded to read the second analog problem for another 5 minutes. After this, subjects were given 10 minutes to generate a summary of both stories, presented side by side in tabular format (See Appendix A for all direct instructions and discovery questions).

In total, Discovery groups were given 20 minutes during the Reading phase, while Direct Instruction groups were given 16 minutes during this phase. Subjects in the discovery groups had the additional task of generating a summary for both the story analogs either individually or in comparison with each other whereas subjects in the direct instruction conditions did not have to generate any summary solutions for the first two analog stories. They were either presented the solution summaries for both analogs together or individually, thus the Discovery groups needed approximately 4 more minutes than the Direct instruction groups during the Reading phase.

In the Solving phase, all students were given 5 minutes to solve a distracter problem, which was not analogous to the stories they studied or summarized (the Birthday problem). This distracter problem was included to discourage ceiling level performance on the transfer task. After attempting to solve the Birthday problem, all subjects were given 5 minutes to solve the Transfer problem (the Tumor Problem) (See Appendix B for complete Birthday and Tumor problems and their solutions). After being asked whether they have previously seen the problems they encountered in this study, students were given research credit for their participation and debriefed about the study's purpose.

Study 2

Method

Participants

This study included 75 Carnegie Mellon undergraduates who received 1 course credit for their participation. Each student was randomly assigned to one of eight conditions, so that there were approximately 9 participants in each condition but due to attrition we were only able to collect complete data for 69 participants. Incomplete data from the 6 participants who chose to drop out of this study was not included in the final data analysis.

Materials

In addition to the materials described in Study 1, subjects read the Bear story which was analogous to the Birthday story either directly after having attempted to solve the Birthday Problem or after solving the Tumor Problem. In total, subjects read two analogous stories (Fire Chief and the General stories), they then summarized or compared the two analog stories according to their condition, after which they attempted to solve one distracter problem (the Birthday Party Problem). Then they proceeded to either solve the distractor analog (Bear problem) or the transfer problem, (the Tumor problem) which was analogous to the first two base analogs, depending on their condition.

Design

This experiment utilized a 2(Compare vs. No compare) x 2(Direct instruction vs. Discovery learning) x 2(Delay vs. No delay) factorial design with Comparison, Instructional method and Delay as between subject factors. The first independent variable (IV) – Comparison, had two levels: Compare vs. No Compare; the second IV, Instructional Method had two levels: Direct Instruction vs. Discovery Learning and the third IV, Delay also had two levels: Delay vs. No delay. The first dependent variable was performance on the Tumor Problem and it was defined by whether or not subjects were able to correctly identify and write out the three components of the convergence solution. The second dependent variable was performance on the Bear problem and it was defined by whether or not subjects correctly mapped the solution of the Birthday problem to the analogous Bear problem.

Procedure

Subjects in all conditions received two analogous stories (the Fire Chief and General stories, in a counterbalanced order), one non-analogous distracter problem (the Birthday Problem), one transfer problem (The Tumor Problem) and a problem analogous to the distracter (the Bear Problem). All three analogous problems (the Fire Chief, General, and Tumor problems) can be solved by dividing an excess resource into smaller units, surrounding a central target, and converging on the target all at once. And the Birthday and Bear analogs require a pendulum solution which has two components: (1) suspending a weight from the available rope/ribbon (2) swinging the rope/ribbon.

Subjects were tested in a quiet testing room on campus and were told that they would be participating in a study about helping literary characters solve problems. After subjects gave their consent, they were told that they would be given five minutes to read and study a story (the General or the Fire Chief story, counterbalanced across participants). The table below outlines what happens across the entire experimental session.

Table 3. Outline of Experiment 2.

Delay	Instruction	Comparison	Read Analog 1	Summarize	Read Analog 2	Summarize	Compare	Solve Distracter	Solve Transfer to first two analogs	Solve transfer to distracter	
No	**Direct**	**No Compare**	Read 5 mins	Study provided summary 3 mins	Read 5 mins	Study provided summary 3 mins	N/A	Solve 5 mins	Solve 5 mins	Solve 5 mins	
No	**Discovery**	**No Compare**	Read 5 mins	Generate summary 5 mins	Read 5 mins	Generate summary 5 mins	N/A	Solve 5 mins	Solve 5 mins	Solve 5 mins	
No	**Direct**	**Comparison**	Read 5 mins	N/A	Read 5 mins	N/A	Study provided comparisons 6 mins	Solve 5 mins	Solve 5 mins	Solve 5 mins	
No	**Discovery**	**Comparison**	Read 5 mins	N/A	Read 5 mins	N/A	Generate comparisons 10 mins	Solve 5 mins	Solve 5 mins	Solve 5 mins	**Solve second transfer to first two analogs**
Yes	**Direct**	**No Compare**	Read 5 mins	Study provided summary 3 mins	Read 5 mins	Study provided summary 3 mins	N/A	Solve 5 mins	Solve 5 mins	Solve 5 mins	Solve 5 mins
Yes	**Discovery**	**No Compare**	Read 5 mins	Generate summary 5 mins	Read 5 mins	Generate summary 5 mins	N/A	Solve 5 mins	Solve 5 mins	Solve 5 mins	Solve 5 mins

Yes	**Direct**	**Comparison**	Read 5 mins	N/A	Read 5 mins	N/A	Study provided comparis ons 6 mins	Solve 5 mins	Solve 5 mins	Solve 5 mins	Solve 5 mins
Yes	**Discovery**	**Comparison**	Read 5 mins	N/A	Read 5 mins	N/A	Generate comparis ons 10 mins	Solve 5 mins	Solve 5 mins	Solve 5 mins	Solve 5 mins

As in Study 1, this experiment can be broken up into two phases: The Reading phase and the Solution generation phase. The Reading phase was different for each subject depending on which of the 8 conditions they were assigned to. The following is an explanation of four of those conditions:

Direct Instruction/No Compare/No Delay: In this condition, students started by reading either the General or Fire Chief problems for 3 minutes. Then, they were asked to read a summary of the problem for 3 minutes. Students then read the second analog problem (either the General or Fire Chief problem) for 3 minutes, and afterwards, studied a summary of the problem for 3 minutes.

Discovery/No Compare/No Delay: In this condition, students initially read the first analog problem (either the General or Fire Chief problem) for 3 minutes. They were then given 5 minutes to summarize the main parts of the story including problem and solution. Students then read the second analog problem (either the General or Fire Chief problem) for 3 minutes, and afterwards, were given 5 minutes to summarize it.

Direct Instruction/Comparison/No Delay: In this condition, students read the first analog problem (either the General or Fire Chief problem) for 3 minutes. Then, they proceeded to read the second analog problem for 3 minutes. After this, subjects were given 6 minutes to study a summary of both stories, presented side by side in tabular format.

Discovery/Comparison/No Delay: In this condition, students read the first analog problem (either the General or Fire Chief problem) for 3 minutes. Then, they proceeded to read the second analog problem for another 3 minutes. After this, subjects were given 10 minutes to generate a summary of both stories, presented side by side in tabular format (See appendix A for all direct instructions and discovery questions).

In total, Discovery groups were given 16 minutes during the Study phase, while Direct Instruction groups were given 12 minutes during the Study phase. In the Solving phase for all 4 No Delay conditions - following the study phase, subjects attempted to solve the Birthday Problem after which they were asked to answer the Bear Problem. They solved the Tumor Problem at the very end. (They were given 5 minutes each to solve the last three problems).

The 4 Delay conditions were different from the No Delay conditions in that after receiving the Birthday Problem they were not asked to solve the Bear Problem directly. They attempted to solve the Tumor Problem before being given the Bear Problem so as to allow further consolidation of knowledge/information from the Birthday Problem (See Appendix B for complete Birthday, Tumor and Bear stories and respective solutions).

After being asked whether they had previously seen the problems they encountered in this study, students were given research credit for their participation and debriefed about the study's purpose.

Results

In order to examine the effect of the Comparison conditions on the rate of solving the transfer problem (Tumor Problem) we first conducted a chi-square test χ^2 (1, N = 102) = 0.36, $p > .05$ which revealed no significant difference between the Compare and No compare conditions (See Table 4.). To make sure that there was no effect of the differences between the two comparison conditions, we conducted a t-test to investigate whether comparison had any effect on the schema quality of solutions generated for the Fire Chief and General story analogs: t (50) = .208, p >.05 and found no significant effect of comparison on schema quality. That is, subjects who compared the two analogs did not generate a better schema than participants who did not. The mean schema quality for the comparison group (M = .80, SD = .05) was approximately the same as the mean schema quality for the no comparison group (M = .81, SD = .05) providing additional evidence for the lack of difference between the schema generated by the two groups.

Table 4. *Note.* Percentages provided in parentheses.

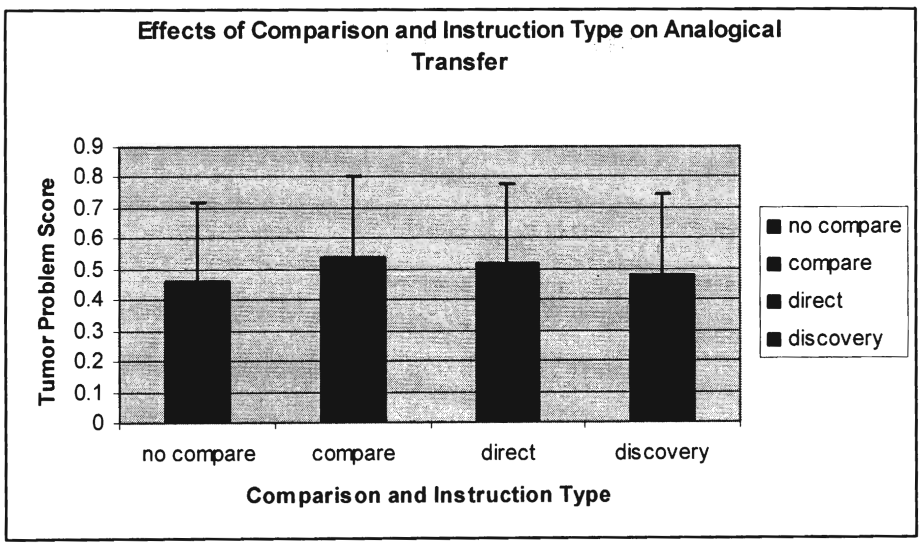
Percentage of Subjects Producing Convergence Solution

	Direct	**Discovery**
Compare	13/25 (**52**)	14/25 (**56**)
No Compare	13/25 (**52**)	12/27 (**44.4**)

There were no differences in the Tumor score of subjects in the Direct and Discovery instructional conditions: χ^2 (1, N = 104) = 0.15, $p > .05$; the mean Tumor problem score for participants in the Direct condition being: 0.60 (SD = 0.42) and the mean score for Discovery participants a close 0.54 (SD = 0.46). We conducted a 2 x 2 ANOVA with

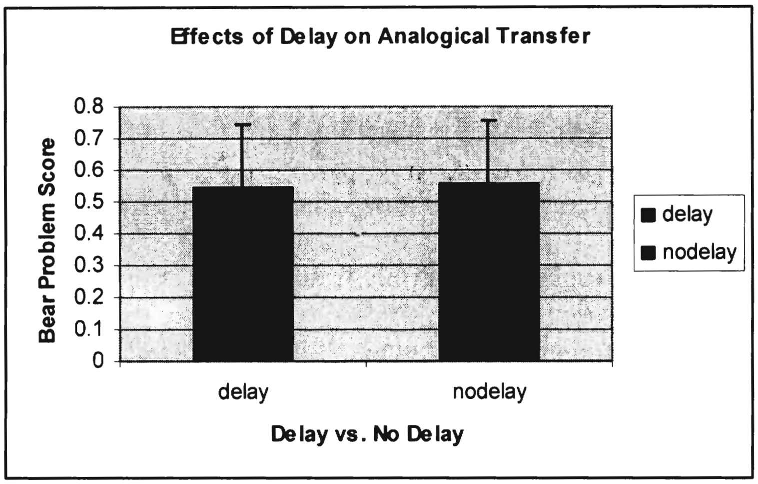
Instruction type and Comparison as between subjects fixed variables and the parametric Tumor score as the dependent variable. We did not find a main effect for Comparison F (1, 106) = .001, $p > .05$ or Instruction type: F (1, 106) = 0.48, $p > .05$, nor did we find an interaction between instruction type and comparison: F (1, 106) = 0.51, $p > .05$ Consistent with our second hypothesis, Direct instruction and Discovery learning had similar effects on participants' ability to solve the Tumor problem. One instructional approach did not result in better analogical transfer than the other (See Figure 5.). We did however find a strong positive correlation between the schema quality generated for the first two story analogs and performance on the Tumor problem: $R^2 = 0.13$. Our finding, that schema quality had a significant effect for solving a transfer problem: F (1, 106) = 7.38, $p < .05$ corroborated previous literature in this area, specifically replicating Gick and Holyoak's (1983) results.

Figure 1.



In order to determine whether Delay helped subjects better solve the transfer problem than no Delay, we compared subjects' scores on the Bear problem and found a marginally significant effect for Delay: t (55) = .97, p = .08. This result supported our third hypothesis with the mean score for the Bear problem in the No delay condition being: 0.32 (*SD* = 0.09) and the mean score in the Delay condition being: 0.45 (*SD* = 0.09). Thus our results indicated that providing a delay prior to solving the Bear problem enhanced performance on the transfer task (See Figure 6.). We conducted an additional regression analysis to explore whether performance on the Birthday problem predicted performance on the Bear problem and found a positive correlation between the two scores: $R^2 = 0.08$. Participants' performance on the Birthday problem had a significant effect on their performance for the analogous transfer task: $F(1, 67) = 5.56, p < .05$ providing strong support for our hypothesis.

Figure 2.



Discussion

Two studies tested the claims that analogical transfer is path independent and a short delay before solving the transfer problem improves participants' ability to correctly map analogical similarities from the base to the target analog. These studies also examined whether comparison between analogs would yield better solution schemas for the analogs and consequently better rates of solving the transfer problem as opposed to no comparison between the analogs. Our results revealed that subjects in the direct instruction condition performed just as well as subjects in the discovery condition on the Tumor problem. Direct instruction did not enable participants to abstract structural commonalities from the analogs and map them to the transfer problem any better than discovery learning did.

This result replicates Klahr and Nigam's (2004) work on the equivalence of instructional paths for analogical transfer in early science instruction. Their research established that third and fourth graders were more likely to achieve mastery of a concept if given direct instructions as opposed to inquiry based instructions, but our study focused only on the effects of instructional methods on analogical transfer, not on learning/mastery. Klahr and Nigam also found that for young children, transfer of scientific knowledge is independent of instructional methods but our study contributes to the literature on the path independence of analogical transfer by providing evidence for the same with a different population. Not only is analogical transfer not dependent on instructional method for young children but also for adults (18-22yr olds). Klahr and Nigam (2004) tested children's analogical transfer in the domain of scientific knowledge whereas we tested undergraduates' analogical abilities in the domain of reading comprehension thus lending to the generalizability of previous research findings to a bigger domain.

While Gick and Holyoak's (1983) research provided evidence for the benefits of guided instructional approaches by showing how direct instruction led to generation of good schemas and subsequently better rates of solving transfer problems, our study does not entirely replicate their findings. Although good schema quality was positively correlated with rates of solving the transfer problem, direct instruction had no advantage over inquiry based instruction. One reason for this effect might be that even though direct instruction makes commonalities between two analogs very salient and promotes the abstraction of relevant schemas (Catrambone & Holyaok, 1989), it fails to engage long-term memory and simulate the unguided context of a transfer situation (Koedinger & Aleven, 2007) which inhibits transfer of relevant knowledge to the target problem. Another reason for students' equivalent performance in both direct and discovery instruction conditions may be that analogical transfer does not depend on instructional type whereas mastery of a concept does (Klahr & Nigam, 2004). Perhaps an additional analysis on the effect of instructional approach on schema quality would have revealed whether one pedagogical approach had an advantage over the other in terms of learning but not transfer.

Our study found that students who compared two analogous examples prior to solving a transfer task did not produce schema any better in quality compared to students who read the examples individually without contrasting them. This finding contradicts previous work done in this area by Gick and Holyoak (1983) as well as Gentner and Namy (2006) among other researchers, who claimed that comparison of analogous examples fosters spontaneous transfer of the convergence schema to the target situation. One possible reason for comparison not having any effect on schema quality is that the wording of our analogs may not have brought to light the relevant structural commonalities between the examples. If the Fire chief and General problem were reworded such that their deep structural similarities were made pertinent and hard to overlook, we might be able to see the benefits of comparison. Rewording the target problem (Tumor problem) such that it emphasized structural features it shared with the source analogs might also have promoted effective abstraction of relevant schema.

Future studies can present similarities between the problem and solution structures in a tabular format so that relevant features easy to compare, diminishing the possibility of abstracting irrelevant commonalities. Including a diagrammatical representation of the convergence solution after the text of the story might help subjects visualize the solution a lot better than just the presentation of text. And recalling the same visualization while reading the second example may help them connect the dots and understand the salient common features of both analogs thus fostering transfer of the convergence solution to a similar situation in the future.

A possible reason for the lack of difference between the schema quality of the comparison and no comparison groups might be that the ability to use examples when they are provided is age sensitive. Even though Gentner et al. (2001) tested undergraduates in an introductory business class and found that students who had learnt a negotiation strategy through comparison of sales examples were better able to apply the same strategy to a later task than students who were not given the opportunity to compare; is it possible that comparison of examples is more beneficial to a specific age group and not everyone? Robust effects for comparison-driven learning have been demonstrated with preschool-age children's abstract categorical reasoning (Kotovsky & Gentner, 1996), word learning (Gentner & Namy 1999; Namy & Gentner, 2002), and learning of animal defense mechanisms (Brown & Kane, 1988).

Our results supported our final hypothesis by providing evidence for the advantage of implementing a delay before solving a transfer task. Subjects who were asked to solve a non analogous problem (Tumor Problem) before being asked to solve a transfer analog were more likely to solve the target (Bear problem) than students who were given the transfer task right after the source analog (Birthday Problem). This finding is consistent with the wealth of literature exploring the effects of unconscious thought on problem solving (Bos et al., 2007; Bargh, Chen & Burrows, 1996; Dijksterhuis & Nordgren, 2006).

Research on unconscious thought thus far has largely been concerned with decisional tasks such as consumer choice and post decisional satisfaction but the present study contributes to the existing literature in this area by providing evidence for the benefits of unconscious thought even for analogical transfer. The distractor problem presented to students before administering the transfer task provided a period of 5 minutes during which their conscious thought was directed away from the Birthday Problem thus delegating the goal of abstracting the deep structure of the source analog to the unconscious. Subjects who were asked to solve the transfer problem right after having solved the source analog were not afforded such an opportunity for unconscious thought and thus were less likely to solve the Bear problem compared to students who were subjected to a delay.

Unconscious thought is not always superior to conscious thought; the relative advantage of the unconscious over the conscious is task- dependent. The unconscious might be better at abstracting schemas from a particular situation but this superiority of unconscious processes does not apply to the initial information acquisition stage in problem solving. The unconscious suffers from poor encoding (Dijksterhuis & Nordgren, 2006) whereas the conscious is well equipped for it and our results echo this "best of both worlds hypothesis" which claims that complex decisions are best made when information is encoded consciously and the later thought process is assigned to unconscious thought. Subjects in the delay condition in the present experiment consciously focused on features of the Birthday Problem which allowed for thorough encoding to occur but the distractor afforded the unconscious to abstract the deep structure of the problem because it confined conscious thought to solving something completely unrelated and non analogous.

Something that we could have changed about the second study is the duration of the temporal delay introduced before the transfer task. Instead of a 5 minute distractor we could have asked participants to come in 2 weeks later and tested whether they were able to solve the analogous problem by mapping relevant commonalities from the previous problem. We were unable to do this because we observed in a trial run that after subjects received credit for coming in for the first session, they were less likely to show up for the second session since they had already fulfilled their credit requirement. Perhaps giving subjects a monetary reward after the completion of their second session would have maintained their interest in our study for a bit longer and reduced the rate of attrition. Needless to say the data from this trial run was not included in our final analysis.

Another change in the study design that could have resulted in a significant effect for delay and a stronger correlation between solution rates for the Birthday and Bear problem is introducing a hint (Catrambone & Holyoak, 1988) prior to having subjects solve the distractor problem. Telling the subjects that they would be asked questions related to the Birthday problem (source analog) at a later time in the session would allow for goal directed unconscious thought even when they were consciously focusing on the Tumor problem (distractor). Dijksterhuis and Nordgren (2006) found that participants who were given the goal to think unconsciously show greater organization and integration than participants who are given a mere distraction without a goal to work toward. Even though participants would be informed that the birthday problem will be relevant for a later task they would not explicitly be told that they will be asked to solve an analogous problem thus not giving away the underlying structure of the study design. In the present study, participants were not given any hint which is why it is possible that they viewed the Tumor problem as a mere distraction and their unconscious thought process was not as organized and coherent as it could have been had they been aware of a goal.

One possible constraint of our study was that due to a limited number of undergraduates taking psychology electives and introductory level classes, only a small percentage of undergraduates qualified to have access to the online sign up system we were using to solicit participants. We ended up testing a total of 108 students, which is not a big enough sample to begin with. Six participants did not complete the study which reduced our sample size to a total of 102 students. Since attrition may have made our student sample less diverse, our study could have had greater reliability had we been able to test students outside of the psychology subject pool.

A single aspect of grading the transfer problem solutions as well as the schema quality for the example analogs, which we could have changed in order to obtain more robust results, is recognizing more than one solution as correct. Though it is important that subjects come up with solutions that share relevant commonalities for both the source analog and the transfer analog, we could accept other analogous pairs of solutions that are different from our expected convergence schema because although subjects may not extract from a problem the features that we want them to, what is important is that they be able to abstract the structure of one analog and apply it to another when they recognize any similarity between the deep structure of the two regardless of what they perceive as the similarity or the components of the convergence solution.

One possible follow up experiment could concern surface similarities of the analogs. Including a salient superficial similarity between two source analogs could direct participants' attention toward the structural similarities of the same analog by indicating the possibility of there being further similarities. Once participants start finding additional surface similarities between the given analogs they may or may not continue their quest for additional similarities and perhaps delve into the deeper relational structure of the same pair of analogs especially if they seem to be running out of surface similarities. Providing just one surface similarity could thus subsequently help foster better abstraction of the convergence solution and map it to the transfer problem resulting in the generation of better schema quality when followed by analogical encoding and also better rates of mapping the relevant features of the analogs to the target problem. A future avenue of research could involve finding ways of helping learners focus on relevant features of training examples and to accurately identify these features in transfer problems (Catrambone & Holyoak, 1988).

Our findings support Klahr and Nigam's (2004) assertion that analogical transfer is independent of instructional method and extend this claim to a population other than 3rd and 4th graders specifically to 18-22 year old undergraduates. Was there something about our sample population that made them learn and transfer knowledge equally well with both pedagogical approaches? Would participants from a different university, city or state have found direct instruction more efficient that discovery learning? What features of a student population make them more suitable for a certain type of instructional method and not another? Additionally is dissemination of knowledge in a certain domain better suited to a particular pedagogy? For example would students be better able to apply principles of physics if their teacher employed direct instruction as opposed to discovery learning? And would inquiry based learning be a better fit for the domain of biology and not math. Such research questions could equip teachers with evidence based guidance and help make their lessons in class more efficient.

Though our study provides evidence against the popular claim that comparison of examples results in adequate transfer to a new situation, many researchers assert that comparison results in benefits that are limited to the concept or principle being taught (Catrambone & Holyoak, 1989; Gick & Holyoak, 1980, 1983; Kurtz et al., 2001; Reeves & Weisberg, 1994). Comparison promotes extraction of schema common to the two cases being studied but does not improve the ability to apply noninstantiated principles to a future problem (Gentner et al., 2003). If comparison limits participants to applying learned principles to analogous situations, then could the lack of comparison have a relative advantage over comparison by promoting the application of concepts that participants have not thoroughly studied or examined examples of?

This study has important implications for the implementation of instructional methods in classrooms at various levels of education (high school, undergraduate and graduate education) although they are primarily generalizable to the undergraduate population in the country. Future research could look into the importance of verbal comparison by asking students to compare two analogs by talking out loud in addition to the textual comparison they first engage in because comparison is applicable to most levels of education including preschool. Could this additional comparison enable participants to engage in more efficient schema abstraction? And can multi-modal assessment of relational structures help students successfully transfer relevant knowledge to a new situation?

Our study paves the way for more research on the effects of pedagogy as well as unconscious thought on analogical transfer but maybe future studies in this area can focus on the factors that determine when the unconscious makes relevant schemas and deep structural commonalities available to our conscious because predictors of when and how the unconscious delivers its ideas to our conscious still remain in the dark. Another largely unexplored idea in the literature on analogical transfer is whether mapping of relational structure from a base analog to a similar situation in real life is easier than transferring the same knowledge to a mundane pencil and paper task. Once again multi-modal assessment of the new situation may either assist or hinder the transfer of knowledge depending on whether the dynamic real life situation distracts a participant from its deep structure by exhibiting unrelated surface features or whether the real life situation aids the mapping of relevant information by involving multiple senses of perception. Such studies would add to the existing literature on educational research and not only enable teachers to teach more efficiently but also assist educational researchers and policy makers in designing school curriculums well suited to specific student populations.

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Appendix A

Source Analogs Used in Studies 1 and 2

The General

A small country was ruled from a strong fortress by an evil dictator. The fortress was situated in the middle of the country, surrounded by farms and villages. Many roads led to the fortress through the countryside. A rebel general vowed to capture the fortress. The general knew that an attack by his entire army would capture the fortress. He gathered his army at the head of one of the roads, ready to launch a full-scale direct attack. However, the general then learned that the dictator had planted mines on each of the roads. The mines were set so that small bodies of men could pass over them safely, since the dictator needed to move his troops and workers to and from the fortress. However, any large force would detonate the mines. Not only would this blow up the road, but it would also destroy many neighboring villages. If the general sent only small troops over the mines, they would be easily repulsed by the defending troops. It therefore seemed impossible to capture the fortress.

The Fire Chief

One day, a very large oil well, which rested at the heart of a major oil plant, exploded and caught fire. The result was a blazing inferno that consumed an enormous quantity of oil each day. The fire was too massive for the plant owners and local firefighters to extinguish. The oil company's owners were very concerned. After initial efforts to put out the fire failed, a notorious Fire Chief was called in to put out the fire. The Fire Chief knew of a new type of retardant foam that would be useful in putting out the fire, and there was enough foam available at the site to do the job. However, there was no hose large enough that could distribute all the necessary foam on the fire fast enough to put it out. The small hoses that were available could not shoot the foam quickly enough to do any good. It looked like there would have to be a costly delay before a serious attempt could be made to extinguish the blazing fire.

Questions for Discovery No Comparison

General Problem Question: Think about the story that you just read. Please describe the problem and the solution of the story.

Ideal Solution: The general considered calling off his attack. Then a strategy to attack the fortress came to him. Instead of sending his full force of troops over the mines, which was certain to end in many casualties, the general divided his forces into smaller units that could pass over the mines safely. Then he had these smaller units surround the fortress. He ordered each smaller unit to march over the mines and attack the fortress, all at the same time. This way, he avoided detonating each mine, but was still able to attack the fortress in full force. The general was able to take the fortress and overthrow the ruling dictator.

Fire Chief Problem Question: Think about the story that you just read. Please describe the problem and the solution of the story.

Ideal Solution: The Fire Chief was not without his wits, however. He immediately took charge and organized everyone. He stationed all his men in a ring around the fire, with all of the available hoses. As soon as the last man was prepared, the chief gave a shout and everyone turned on their hoses and foam was directed at the fire from all directions. In this way, a huge amount of foam quickly struck the source of the fire, and it was soon extinguished and brought under control. The Fire Chief held up his famed fire fighting reputation that day.

Question and Answer for Discovery Comparison

Problem: Think about the two stories that you just read. In particular, think about the similarities between these two stories. What are the key parallels? Please describe the problems and solutions of both stories, making sure to note their commonalities.

Key Parallels: The main goal of the general and fire chief is similar because in both cases they want to focus a large force (soldiers/water) on some target (the fortress/ the fire) that cannot be used in full strength from a single direction. The protagonist in both cases splits his resources into smaller units, surrounds the problem, and converges all at once.

Direct No Comparison Instructions

The Fire Chief

Think about the story that you just read. Please study this summary of the story's problem and solution:

The Fire Chief's goal was to extinguish a massive fire that had erupted at a major oil plant. There was an abundance of fire-fighting retardant foam, hoses, and men to fight the fire: *There was an abundance of resources with which to eliminate a problem.*

However, none of the hoses were large enough to distribute the foam at a sufficient rate to extinguish the fire: *Something prevented the protagonist from using his resources to full capacity*

The Fire Chief stationed his men in a ring around the fire and each man was equipped with a small hose. Every man surrounding the fire directed the hoses all at once at the center of the fire. The convergence of many small hoses all at once was enough to put out the fire: *The protagonist splits his resources into smaller units, surrounds the problem, and converges all at once.*

The General

Think about the story that you just read. Please study this summary of the story's problem and solution:

The General's goal was to attack and overthrow a fortress inhabited by an evil dictator and the General had many soldiers at his command: *There was a large amount of resources available capable of destroying a target.*

However, mines were placed along the roads, so the General's large army could not reach the fortress. While small units of soldiers could safely cross the mines, they would be easily repelled by the dictator: *There was something that stopped the protagonist from maximizing his resources' full potential*

The General split up his army into smaller units and encircled the fortress. The General then ordered his men to converge on the fortress, all at once. By surrounding the fortress with smaller units and attacking at the same time, the General was able to overtake the fortress*: The protagonist divides his resources into lesser units, encircles the target, and directs them simultaneously at the target*

Direct Comparison Instructions

Think about the two stories that you just read. Please study this summary of each story's problem and solution. In particular, pay close attention to the similarities between these two stories, keeping in mind the key parallels between them:

The General's goal was to attack and overthrow a fortress inhabited by an evil dictator and the General had many soldiers at his command.

The Fire Chief's goal was to extinguish a massive fire that had erupted at a major oil plant. There was an abundance of fire-fighting retardant foam, hoses, and men to fight the fire.

In both stories, there was an abundance of resources with which to eliminate a problem.

However, mines were placed along the roads, so the General's large army could not reach the fortress. While small units of soldiers could safely cross the mines, they would be easily repelled by the dictator.

However, none of the hoses were large enough to distribute the foam at a sufficient rate to extinguish the fire

In both stories, something prevented the protagonist from using his resources to full capacity

The General split up his army into smaller units and encircled the fortress. The General then ordered his men to converge on the fortress, all at once. By surrounding the fortress with smaller units and attacking at the same time, the General was able to overtake the fortress.

The Fire Chief stationed his men in a ring around the fire and each man was equipped with a small hose. Every man surrounding the fire directed the hoses all at once at the center of the fire. The convergence of many small hoses all at once was enough to put out the fire.

In both stories, the protagonist splits his resources into smaller units, surrounds the problem, and converges all at once

Appendix B

Transfer Problem for Study 1 and Distractor for Study 2

The Tumor Problem

Story: One day, a doctor was faced with a very difficult problem. A malignant tumor had developed in the stomach of one of his patients. If the tumor weren't treated soon, the cancer would spread throughout the patient's body, resulting in death. Because of some medical complication it was impossible to perform an operation to remove the tumor or restrict its blood supply. There was, therefore, no simple way to treat the patient's condition. However, the doctor had available to him a new kind of laser that the hospital had just bought for each of its doctors. The laser was capable of shooting a ray that could be used to destroy the tumor. The doctor knew that a sustained large dose of the ray would effectively destroy the tumor. Unfortunately, the ray would also destroy the healthy tissue that it would pass through on the way to the tumor. At a lesser dosage, the ray would not harm the healthy tissue, but it would not destroy the tumor either.

Problem Question: How can the doctor destroy the tumor with the ray, and at the same time avoid destroying the healthy tissue? Suggest as many possible solutions as you can. Write down all the possibilities you can think of, even ones that may not really be practical. Don't worry about not having enough medical knowledge.

Ideal Solution: The doctor was troubled while he pondered this problem, but then thought of an idea that might work. He considered gathering together many of the lasers in the hospital, and focusing their target on the spot of his patient's tumor. However, instead of using the full force of each laser, the doctor would spread the lasers out so that each ray would enter the patient's body at a different point, while still converging on the tumor. He would also set the dosage of the ray to low, so that each ray would not destroy any healthy tissue, but would destroy the tumor when low dosages of the rays converged. The doctor tried this and was successful in eradicating the tumor.

Source Analog for Study 2

The Birthday Problem

Story: It was Jane's 6^{th} birthday and her mother, wanting to make the day very special for Jane, organized a big party for her. Jane's mother had all the food prepared for the party and all of Jane's friends had been invited, but just before the guests were supposed to arrive, Jane's mother was still finishing up a decoration pattern. Two final pieces of ribbon were left that were dangling from the wood paneling above. She had originally planned to knot these two pieces of ribbon together in order to attach balloons to them. However, whenever she grabbed the end of one ribbon (the blue ribbon), she was not able to grasp the other ribbon (the pink ribbon) at the same time. They hung from two different parts of the room, and Jane's mother's arms simply were not long enough to reach both ribbons at once. It seemed that she might have to abandon her final bit of decoration.

Problem Question: Can you think of any way that Jane's mother can reach the other ribbon in order to tie the two ribbons together? Suggest as many possible solutions as you can. Write down all the possibilities you can think of, even ones that may not really be practical. Use any information you can think of to help solve the problem.

Ideal Solution: Jane's mother was just about to give up, when she had an idea. She took the pair of scissors that she had been using to cut the various ribbons and attached the scissors to the end of the pink ribbon. Next, she took hold of the scissors, and walked as close to the blue ribbon as possible. She then let the scissors go and ran as quickly as she could to the dangling blue ribbon. She then waited until the swinging pink ribbon came her way and caught it on the upswing. While still holding the blue ribbon, she then removed the scissors from the pink ribbon and knotted the two ribbons together. Just after this, the guests arrived, and Jane had a great time at her birthday party.

Transfer Problem for Study 2

The Bear Problem

Story: Kevin and Joshua are on their way back from a hiking trip in the mountains and suddenly hear a noise behind them, it sounds like running footsteps but when they stop and look, they see a bear charging at them. The friends run frantically until they reach a rickety bridge. They both know that only one person can cross the bridge at a time or else it will collapse, so Kevin sprints across the bridge and makes it safely to the other side but unfortunately the safety line (rope) that is under the bridge snaps from the middle and half of the bridge on Kevin's side collapses. Joshua can run up to the middle of the bridge but cannot go any further; he will be eaten by the bear unless Kevin finds a way to rescue his friend.

Problem Question: What can they do to get Josh over to the other side? Suggest as many possible solutions as you can. Write down all the possibilities you can think of, even ones that may not really be practical. Use any information you can think of to help solve the problem.

Ideal Solution: After Kevin safely crosses the bridge to the other side he finds a rock/ his own shoe/ one of the wooden planks that came off the bridge when it collapsed to tie to his end of the rope and swings it like a pendulum to Joshua's side who is waiting at the very middle of the bridge close to where it had collapsed. Joshua catches the rope that Kevin swings toward him and ties it to his end of the rope. He then shimmies across the safety line toward Kevin's side, far away from the bear.

Note: The Birthday Problem and the Bear problem were both presented along with hand-drawn figures that diagrammatically represented each scenario aiding subjects in their comprehension of these problems. Since base-line analysis revealed that these problems were very difficult to solve, we included these figures to make it easier for subjects to visualize both situations and to avoid a floor effect for solving.